

Inductive Reasoning // Counterexamples

Lesson Objective

INBAT make and test conjectures using inductive reasoning; find counterexamples to prove statements false.

Conjecture: A conjecture is an unproven statement based on observations.

Inductive Reasoning: when you find a pattern in specific cases and then write a conjecture for the general case.

Example:

1. Sketch the next figure in the pattern



2. Sketch the next figure in the pattern



Making and Testing a Conjecture

Example:

3. Numbers such as 3, 4, and 5 are called consecutive integers. Make a test a conjecture about the sum of any three consecutive integers.

$$4 + 5 + 6 = 15 = 5 \cdot 3$$

$$1 + 2 + 3 = 6 = 2 \cdot 3$$

$$7 + 8 + 9 = 24 = 8 \cdot 3$$

$$20 + 21 + 22 = 63 = 21 \cdot 3$$

* 3 times the second (or middle) number.

Test:

$$13 + 14 + 15 = 42 = 14 \cdot 3$$

$$42 = 42 \checkmark$$

4. Make and test a conjecture about the product of two odd integers.

Counterexamples

To show a conjecture is true, you must show that it is true for ALL cases.

You can show it is false, however, by finding just ONE counterexample.

A counterexample is a specific case for which the conjecture is false.

Example:

5. A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

"The sum of two numbers is always greater than their difference."

$$(-3) + (-4) = -7$$

$$(-3) - (-4) = 1 \quad \leftarrow \text{diff. is greater!}$$

6. Find a counterexample to show that the conjecture is false.

"The value of x^2 is always greater than the value of x ."

$$1^2 = 1$$

$$1 \not> 1$$

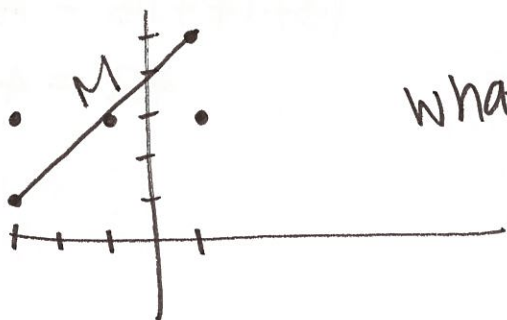
7. Find a counterexample to show that the conjecture is false.

"If $1 - y > 0$, then $0 < y < 1$."

$$1 - (-2) > 0 \quad \text{but} \quad 0 \not< -2 < 1$$

8. Determine if each biconditional is true. If false, provide a counterexample.

"A segment has endpoints at $(1, 5)$ and $(-3, 1)$ if and only if its midpoint is at $(-1, 3)$."



what about $(-3, 3)$ and $(1, 3)$
as endpoints? Mid is still $(-1, 3)$!